The Viscosity of Liquid Metals

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In order to make the kinetic theory of liquid viscosity tractable, an approximate form is suggested for the perturbation of the radial distribution function of a monatomic liquid by a nonuniform flow field. Substitution of this form into the microscopic expression for the pressure tensor yields an equation for liquid viscosity in terms of the equilibrium distribution function and the interatomic potential energy function. This equation establishes the basis for a corresponding states correlation of the viscosity of liquid metals based on atomic parameters.

The viscosity data for twenty-one molten metals are made to fall on a single curve by the adjustment of one microscopic parameter. It is found that this empirically determined parameter has the proper fundamental significance. Therefore it is possible to estimate it independently and to use the general correlation for estimating the viscosity of a metal for which data are not available. Also it is suggested that the atomic parameters determined from viscosity data in this way might be used to correlate other properties of the liquid metals.

In recent years a great deal of effort has been devoted to the measurement of the viscosity of molten metals. One reason for this is that the behavior of the viscosity of simple liquids is a vital key to understanding the nature of the liquid state. On the practical side these data are of interest because of the increasing application of liquid metal coolants in nuclear reactors. Also, the viscosities of many higher melting metals are required for design purposes in the metallurgical industries. For example, in the steel industry the recent development of continuous casting processes requires an accurate knowledge of the viscosity of the molten metal. It is obvious that the need for such data for less common metals and particularly for molten alloys will increase. Therefore, because of the experimental difficulties of measuring properties at such high temperatures, it would be desirable to have a reliable correlation from which the viscosity of a particular substance could be estimated. The purpose of this paper is to show how the existing theory of liquid structure can be treated to develop a useful engineering correlation of the viscosities of liquid metals.

When seeking a meaningful correlation of a physical property such as viscosity, we desire to base our analysis upon a sound theory rather than to rely entirely on empiricism or artificial models. In this way we hope to obtain relationships and derived parameters which have real physical significance. With such a correlation one has confidence in extrapolating existing data over a range of conditions or in predicting values for substances where no experimental data are available. For example, the equations of statistical mechanics and kinetic theory have been applied very successfully to correlate the thermodynamic and transport properties of gases (*1*). The liquid state, however, does not lend itself so readily to theoretical analysis. In general most theories of liquids have been based on models which generate parameters lacking fundamental significance and which impose a priori assumptions on the structure, interactions, or mechanisms of transport in the fluid. The theory of liquids based on the pair distribution function as developed by Kirkwood (*2*) and by Born and Green (*3*) escapes this criticism because of the generality of its formulation. In fact, all the thermodynamic and transport properties of a fluid can be expressed with general validity in terms of the intermolecular potential energy function and the appropriate distribution functions. The chief stumbling block in applying this theory lies in obtaining valid representations of these distribution functions.

Over the past ten years some excellent work has been done on extending these theories to make possible the direct calculation of the properties of simple fluids in the high density region. Noteworthy is the work of Rice, Gray, and co-workers (*4* to *6*) and Longuet-Higgins (*7*). [See, for example, the recent review by Suddaby (*8*).] Although this work is quite elegant and leads to important and significant results, it is unfortunate from the standpoint of the engineer that the physical picture, which is clear in the formulation of the problem, it necessarily obscured in the subsequent mathematical development. It is probably for this reason that the hole theory and the activated state theory of transport processes have maintained their popularity with many engineers and physical chemists; although these models are not physically valid they do offer a simple intuitive picture that one can easily apply. This paper deals with the fundamental physics of the phenomenon, which have been described by Kirkwood and Born and Green, and attempts to develop a useful intuitive model which, however, is consistent with the actual situation. The advantage of this approach over the simpler models is that in addition to maintaining a physical picture of the phenomenon it also leads to a theoretically meaningful correlation of liquid metal viscosity data.

THEORY

According to the Born and Green theory (*3*), the pressure tensor P in a monatomic fluid can be expressed as

$$P = m \int\int\int \widetilde{\xi}\,(\widetilde{\xi} - u)\, f\,(\widetilde{\xi})\, d^3\widetilde{\xi} - \frac{n^2}{2} \int\int\int \mathbf{r}\, \nabla\phi(r)\, g(\mathbf{r})\, d^3\mathbf{r} \tag{1}$$

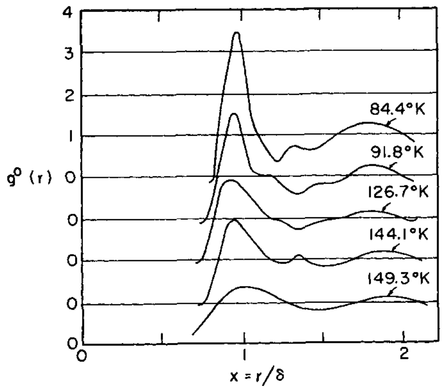

Fig. 1. The experimental radial distribution function for liquid argon at various temperatures (*10*).

The first term in this expression corresponds to the kinetic contribution to the flux of momentum, that is, the momentum which is transported by the actual movement of molecules through the fluid. It is this term which is considered in the kinetic theory gases as presented by Chapman and Cowling (*1*).

In a nonuniform liquid however, most of the momentum is transported by intermolecular forces, and this contribution to the pressure tensor is given by the second term in Equation (1). In effect, this integral adds up the interactions between all pairs of atoms. The quantity $\frac{n^2 g(\mathbf{r})}{2}$ is the time-average number of pairs with a separation $\mathbf{r}$.

At equilibrium the pair distribution function $g(\mathbf{r})$ is symmetrical and is known as the radial distribution function $g^o(r)$. This quantity can be measured experimentally by x-ray or neutron diffraction techniques (*9*). Shown in Figure 1 are experimental curves representing the radial distribution function g^o of liquid argon at various temperatures which were measured by Eisenstein and Gingrich (*10*). The curves have been vertically displaced to show the effect of temperature more clearly. In each case $g^o(r)$ approaches unity at large separations which means that the long-range structure is random. The distance coordinate has been normalized by a length δ, the distance to the first peak which is also approximately the position of the minimum of the pair potential energy function ϕ.

A significant characteristic of the radial distribution function is the effect of temperature as demonstrated by Figure 1. It is seen that increasing temperature causes the function g^o to tend toward unity at all positions along the curve. This means that the increased thermal motion of the particles tends to destroy the order in the liquid at all separations. This effect is found to occur even at constant volume as is shown by the recent data of Pings (*11*).

Under conditions of thermal and mechanical equilibrium when the pair distribution function g is equal to the symmetrical function g^o, the shear stresses in the pressure tensor vanish and Equation (1) reduces to the equation of state (*12*):

$$p = nkT - \frac{2\pi}{3} n \int_0^\infty r^3 \frac{d\phi(r)}{dr} g^o(r)\, dr \qquad (2)$$

When the fluid is not in equilibrium, the macroscopic pressure tensor includes certain off-diagonal elements as well as the pressure defined by Equation (2), and it can be written as (*13*)

$$P = pI + \tau \qquad (3)$$

where I is the unit tensor and τ is the stress tensor. Substitution of Equations (1) and (2) into Equation (3) leads to a microscopic expression for the pressure tensor

$$\tau = \tau_K - \frac{n^2}{2} \int\int\int \mathbf{r} \frac{d\phi(r)}{dr} g'(\mathbf{r})\, dV \qquad (4)$$

where τ_K is the kinetic contribution to τ and the second term represents the contribution of intermolecular forces. Since the thermodynamic pressure p includes the equilibrium form of the pair distribution function, the function which appears in Equation (4), $g'(\mathbf{r})$, is the difference between $g(\mathbf{r})$ and $g^o(r)$. That is, the function $g'(\mathbf{r})$ is the deviation of $g(\mathbf{r})$ from its equilibrium value due to nonuniformity in the liquid. The magnitude of the intermolecular force portion of the shear stresses depends directly on the extent of this deviation.

According to the Navier-Stokes stress argument for an incompressible Newtonian fluid (*14*) the coefficient of shear viscosity η is defined as the constant of proportionality between the stress tensor and the deformation of the velocity $[\nabla\mathbf{u} + (\nabla\mathbf{u})\dagger]$, where $(\nabla\mathbf{u})\dagger$ is the transpose of the gradient of the velocity $\nabla\mathbf{u}$:

$$\tau = -\eta\, [\nabla\mathbf{u} + (\nabla\mathbf{u})\dagger] \qquad (5)$$

By comparing this macroscopic definition of viscosity with the microscopic expression for the stress tensor, Equation (4), and by neglecting τ_K since it is very small in a liquid, one obtains a formal expression for the viscosity of a monatomic liquid:

$$\eta[\nabla\mathbf{u} + (\nabla\mathbf{u})\dagger] = \frac{n^2}{2} \int\int\int \mathbf{r}\, \nabla\phi(r)\, g'(\mathbf{r})\, dV \qquad (6)$$

This form is essentially equivalent to the expression given by Born and Green (*15*), except for the omission of the kinetic term. This approximation can be justified by calculating the kinetic contribution to viscosity from liquid self-diffusion data. For example, the self-diffusion coefficient of a liquid metal is on the order of 10^{-5} sq. cm./sec. According to Kirkwood (*16*) the kinetic portion of viscosity η_K is related to the self-diffusion coefficient D by

$$\eta_K = \frac{n\, m}{2} D \qquad (7)$$

For mercury at 300°K., $\eta = 1.55$ centipoise and $\eta_K = 0.02$ centipoise. Thus it is seen that η_K is on the order of only a few percent of the total viscosity. This magnitude is similar to that which Lowry, Gray, and Rice calculated for the kinetic contribution to the viscosity of saturated liquid argon (*5*).

In order to proceed to the calculation of viscosity from Equation (6), it is necessary to obtain the pair distribution function under nonequilibrium conditions and in particular to relate it to the velocity gradient in the fluid. This problem seems to be far from an accurate solution since it is not yet possible to calculate the distribution function of a liquid in equilibrium very precisely. However, it is possible to derive a simple approximation to the exact function by taking recourse to physical and mathematical arguments about the effect of a flow field on the liquid structure.

In general the exact solution for the pair distribution function must be determined from a complicated integro-differential equation (*17*) which is difficult to solve even in the simplest case (*18*). In order to obtain an approximate form for the pair distribution function we consider the equilibrium solution to be known and assume that the general solution to the complete equation can then be expressed as a perturbation series expansion about the equilibrium value, that is

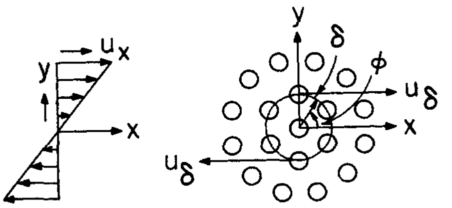

Fig. 2. Schematic diagram for a linear velocity gradient in a monatomic liquid shown on the macroscopic and microscopic scales. u_δ is the average velocity of an atom on the sphere $r = \delta$ relative to that of the central atom.

$$g(\mathbf{r}) = g^o(r) + F_1(r)s + F_2(r)s^2 + \ldots \quad . \qquad (8)$$

The function $g(\mathbf{r})$ is written as a power series in some perturbation parameter s which characterizes the disturbance of the liquid structure by the flow field. s should be small and vanish at equilibrium. The coefficients $F_i(r)$ are taken to be independent of the disturbance and functions of r only. Then s must be a function of the polar angles θ and ϕ. This separation of variables seems reasonable since the disturbance must depend on the velocity gradient, and in the region where interatomic forces are appreciable the velocity gradient is very nearly linear. Therefore s should not depend on the magnitude of the position vector $\mathbf{r}$ but only on its direction.

According to conservation of mass, the average value of $g(\mathbf{r})$ is unity as is the average value of $g^o(r)$. Therefore the integral of each higher term in Equation (8) over r must vanish. This condition will be satisfied if $F_i(r)$ is proportional to $[1 - g^o(r)]$. Thus it is assumed that

$$F_i(r) = A_i\,[1 - g^o(r)] \qquad (9)$$

The factors A_i are constants of integration which would have to be evaluated from the actual solution of the complete equation for $g(\mathbf{r})$. We shall not concern ourselves with their actual values at this time.

An appropriate form of the perturbation parameter s is assumed to be the ratio of two velocities: the ratio of a characteristic disturbing velocity u_δ to a characteristic restoring velocity u^*

$$s = \frac{u_\delta}{u^*} \qquad (10)$$

The velocity which acts to restore the equilibrium liquid structure is assumed to be the average thermal speed of the atoms

$$u^* = \sqrt{3kT/m} \qquad (11)$$

The velocity u_δ is chosen to be the average speed of the atoms at a distance δ from a particular atom, where δ is the atomic diameter. Therefore u_δ is approximately the average velocity of the atoms in the first peak of the radial distribution function relative to that of the central atom. Figure 2 shows the significance of u_δ for the simple case of a linear velocity profile.

In this situation the deformation of the velocity contains only two nonzero elements, each of which is the velocity gradient $\dfrac{du_x}{dy}$:

$$[\nabla \mathbf{u} + (\nabla \mathbf{u})\dagger] = \begin{bmatrix} 0 & \dfrac{du_x}{dy} & 0 \\ \dfrac{du_x}{dy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (12)$$

The macoscopic picture of this flow field is shown on the left in the figure. The velocity u_x is drawn relative to that of the plane $y = 0$. On the right the same situation is shown microscopically. Since we are considering a time-averaged picture u_δ is zero for the atoms in the $y = 0$ plane. However, all other atoms on the spherical surface $r = \delta$ are on the average moving relative to the central atom because of the imposed u_x field. If one imagines an atom on $\mathbf{r} = \delta$ to move away from its equilibrium position with velocity u_δ, one sees that there will be a density deficiency in that region for a short period of time, thus diminishing the value of g. However, because of the random motion of the atoms, as one atom is forced to move away from an energetically favorable region another will shortly replace it with the relaxation time depending on the thermal motion characterized by $\mathbf{u}^*$. Thus the ratio assumed in Equation (11) can be considered to be the function which characterizes the steady state disturbance of the short-range order in the liquid by the macroscopic velocity field. For this simple geometry u_δ can be expressed as

$$u_\delta = \delta \left(\frac{du_x}{dy} \right) \sin\phi \qquad (13)$$

where ϕ is the angle measured from the plane $y = 0$.

Since s turns out to be on the order of 10^{-12} one would expect terms higher than linear in s in Equation (8) to be negligible under ordinary circumstances. If such is the case, the function $g(\mathbf{r})$ can then be approximated by

$$g(\mathbf{r}) = g^o(r) + A_1\,[1 - g^o(r)]s \qquad (14)$$

This approximate form seems physically reasonable. Since the correction to $g^o(r)$ is proportional to $[1 - g^o(r)]$, an increase in s causes $g(\mathbf{r})$ to tend toward unity. That is, as the disturbances become greater, the structure of the liquid becomes more random, an effect similar to that of increasing temperature.

It should be noted that the assumptions incorporated in the form of Equation (14) are consistent with the mathematical requirements of the perturbation function as described by Born and Green (3).

Substitution of the assumed form of the perturbation into Equation (6) yields an expression of liquid viscosity

$$\eta = A_1 \frac{\pi n^2 \delta}{u^*} \int_0^\infty r^3 \frac{d\phi(r)}{dr}\,[1 - g^o(r)]\,dr \qquad (15)\ddagger$$

Thus it is seen that the foregoing assumptions lead to an equation which involves the undetermined constant A_1 as well as the pair potential function $\phi(r)$, and the radial distribution function $g^o(r)$ which are also unknown. Nevertheless a useful conclusion can be drawn from Equation (15). Although the exact nature of the forces between atoms in a liquid metal has not yet been elucidated, it seems reasonable to assume that they can be described to a fair approximation by a potential function $\phi(r)$ which depends on two parameters: an energy parameter ϵ and the distance parameter δ. On the basis of this assumption Equation (15) can be made dimensionless to define a reduced viscosity which is a function of a reduced temperature T^* and a reduced volume V^*:

$$\eta^* = F\,(T^*, V^*) \qquad (16)$$

where

$$\eta^* = \left[\frac{\eta\,\delta^2\,N_o}{(MRT)^{1/2}} \right] \qquad (17)$$

$$T^* = [kT/\epsilon] \qquad (18)$$

$$V^* = [1/n\delta^3] \qquad (19)$$

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‡ It is generally recognized (5) that in the condensed phase allowance must be made for a hard core of the atom such that the lower limit of integration is actually a finite distance; the integral is then convergent.

Furthermore, for a class of substances which obey the same potential energy function $g^o(r)$ should be a universal function of T^*, V^*, and $x = r/\delta$, and the reduced viscosity η^* should be a universal function of T^* and V^*.

The dependence of the radial distribution function of a liquid metal on density has not been investigated. However, Pings' experiments (*11*) with liquid argon below the critical point indicate only a slight sensitivity of $g^o(r)$ to density even at high pressures. In the case of the metals only low pressures are encountered normally, and one would not expect $g^o(r)$ to depend very greatly on V^*. In fact the state of the metal is very close to that of saturated liquid where the relationship between density and temperature is uniquely determined. Therefore, according to Equation (15), a good approximation ought to be that $\eta^*(V^*)^2$ is a universal function of T^* only:

$$\eta^* (V^*)^2 = G(T^*) \tag{20}$$

where

$$G(T^*) = \frac{C_1}{T^*}\int_o^\infty x^3\left[\frac{d(\phi(x)/\epsilon)}{dx}\right][1 - g^o(x, T^*)]dx \tag{21}$$

and C_1 is a constant.

A corresponding states principle of this sort can be established in a number of ways as, for example, by the general development by Helfand and Rice (*19*). However, for the case of the liquid metals the formulation presented here is useful because it provides the explicit density dependence of η^*.

APPLICATION OF THE THEORY TO LIQUID METALS

In order to apply the results of the preceding analysis to develop a useful correlation of the viscosities of liquid metals, it is necessary to have values for the atomic parameters ϵ and δ. Unfortunately there is little evidence to indicate the form of $\phi(r)$ for metals, not to mention the actual values of the parameters. Determination from thermodynamic data such as heat of sublimation is complicated by the effects of the free-electron metallic binding in the condensed phase. Nevertheless, to establish a corresponding state plot of the function $\eta^*(V^*)^2$ vs. T^* it is necessary only to assume that these substances all obey the same function $\phi(r)$ and then to choose some basis for relative values of ϵ and δ.

Ling (*20*) has determined effective Lennard-Jones parameters for sodium and potassium from their experimental x-ray scattering curves. These parameters for the two metals have been used to plot their viscosity data in Figure 3 as $\eta^*(V^*)^2$ vs. $1/T^*$. The ranges of temperature represented here are 350° to 650°K. for potassium and 400° to 1,000°K. for sodium. The viscosity range is 0.605 to 0.165 centipoise for sodium, 0.514 to 0.010 centipoise for potassium, and the reduced volumes at a T^* of 0.25 are 0.80 for potassium and 0.75 for sodium. It is seen that the data plotted in this way fall on a single line as predicted by the theory and that neglect of the implicit dependence on V^* is apparently a good approximation.

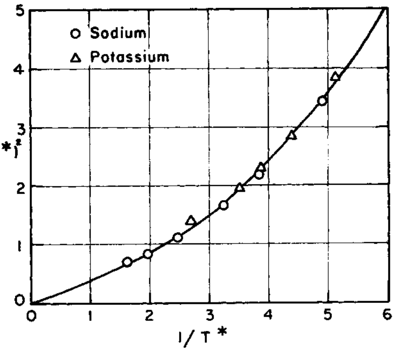

Fig. 3. The reduced viscosities of sodium and potassium as functions of reduced temperature. The reduced quantities have been calculated on the basis of the atomic parameters determined by Ling (*20*).

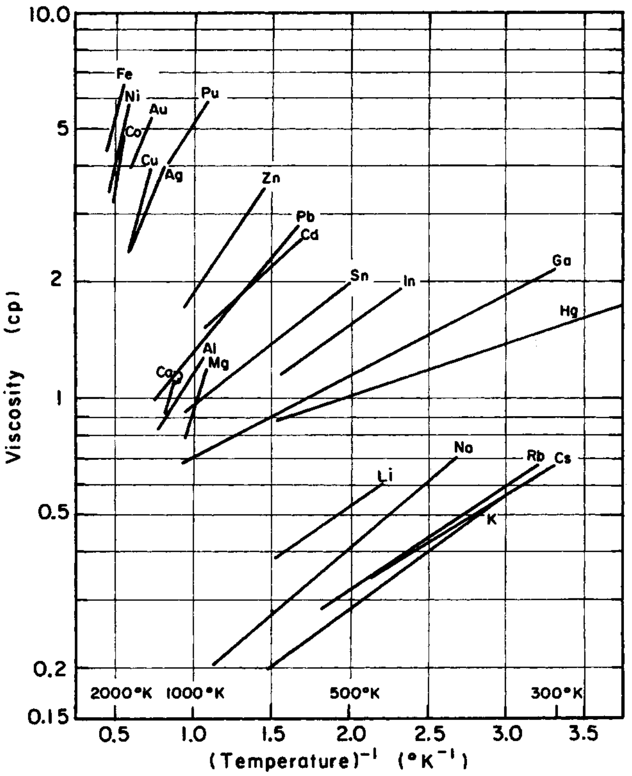

Fig. 4. The viscosities of liquid metals and their dependence on temperature.

The viscosities of many other liquid metals have been measured over a wide range of temperatures. The available data are shown in Figure 4. It is seen that the temperature range is nearly 2,000°K. and that the viscosities range over nearly two orders of magnitude.

TABLE 1. GOLDSCHMIDT ATOMIC DIAMETERS AND ENERGY PARAMETERS OF THE METAL

Metals	δ(Å) (*21*)	ϵ/k, °K.
Na	3.84	1,970 (*20*)
K	4.76	1,760 (*20*)
Li	3.14	2,350
Mg	3.20	4,300
Al	2.86	4,250
Ca	4.02	5,250
Fe	2.52	10,900
Co	2.32	9,550
Ni	2.50	9,750
Cu	2.56	6,600
Zn	2.74	4,700
Rb	5.04	1,600
Ag	2.88	6,400
Cd	3.04	3,300
In	3.14	2,500
Sn	3.16	2,650
Cs	5.40	1,550
Au	2.88	6,750
Hg	3.10	1,250
Pb	3.50	2,800
Pu	3.1	5,550

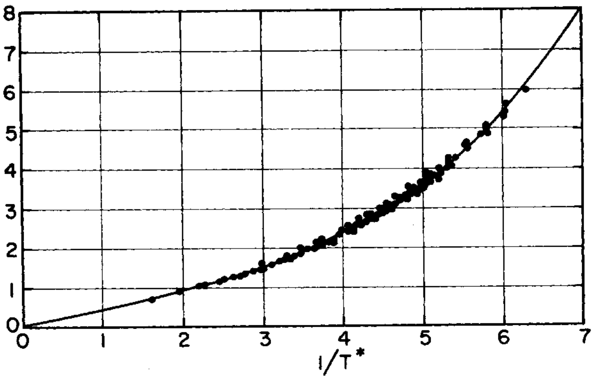

Fig. 5. The reduced viscosities of liquid metals and their dependence on reduced temperature.

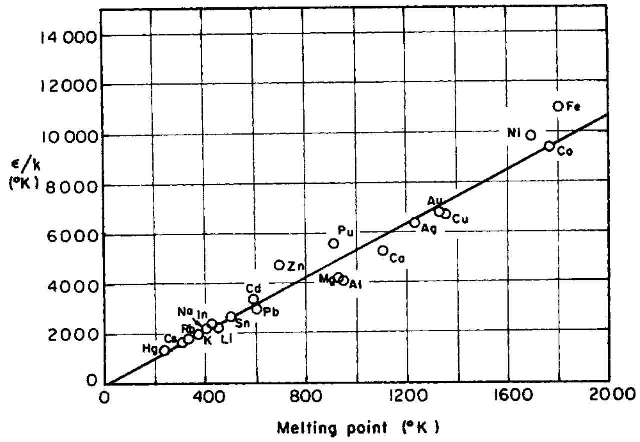

Fig. 6. The correlation of the energy parameter ε with melting point for the liquid metals. The line represents the relationship $\epsilon/k = 5.20\,T_m$.

Since the parameter δ is supposed to be the atomic diameter, it should be possible to estimate its value from crystallographic data. Therefore we have chosen the value of δ to be the interatomic distance in the close-packed crystal of 0°K. These values are known for nearly all metals (*21*). With this estimate for δ, $\eta^*(V^*)^2$ for all the data in Figure 4 can be calculated. Then from the general $\eta^*(V^*)^2$ vs. $1/T^*$ curve, which is established by the data for sodium and potassium, it is possible to evaluate ε for the rest of the metals. That is, we adjust ε to fit the data to the empirical curve in Figure 3. This has been done, and the resulting correlation is shown in Figure 5. The values of the parameters are given in Table 1.

In Figure 5 are plotted the data for the twenty-one different metals, from lithium and mercury to iron and plutonium, with a range of densities from 1.8 to 18 g./cc. and atomic weights from 6.9 to 242. It is seen that this method of plotting the data accounts very well for the dependence of liquid viscosity on the physical variables. The viscosities of liquid metals do indeed obey a corresponding states law.

However, in spite of the fact that the data can be reduced to a single curve, one might wonder whether this plot has fundamental significance because the parameter ε has been determined empirically. If the theory presented here is valid, then the values of ε obtained should in fact be the energy characteristic of the atomic interaction and should be related to other physical properties. Normally one finds that ε for a class of substances correlates with the critical temperatures, but unfortunately these are not known for most of the metals. However, various theories of melting such as that of Lennard-Jones and Devonshire (*22*) indicate that the melting point should be proportional to ε. Such a relationship has been observed for other classes of substances (*22*).

In Figure 6 the values of ε determined here are plotted vs. the melting points, and it is seen that a very satisfactory correlation exists. On this basis, it is concluded that ε does in fact have the proper physical significance. Thus it should be possible to estimate ε for other metals from their melting points in order to predict viscosity from Figure 5.

At the time that the correlation presented here was developed no data were available for the viscosity of liquid thallium. Very recently, however, Cahill and Grosse have reported values for the viscosity of thallium over a wide range of temperature (*24*). In order to demonstrate the procedure for using the correlation as well as to check the accuracy of its prediction in this case we shall compare the estimated viscosity with the experimental values.

For thallium the atomic weight M is 204.4, the atomic diameter δ is 3.40 Å. (*13*), and the melting point is 576°K. The relationship between the parameter ε and melting point represented by the line in Figure 6 can be expressed as

$$\frac{\epsilon}{k} = 5.20\,T_m \tag{22}$$

Therefore the energy parameter for thallium is estimated to be (3,000 k) °K. With this quantity known the reduced temperature corresponding to any absolute temperature can be calculated from Equation (18). For example, at $T = 644$°K., $T^* = 0.214$, and $1/T^* = 4.66$. From Figure 5 it can be determined that the reduced viscosity function $\eta^*(V^*)^2$ corresponding to this reduced temperature is 3.05. At $T = 644$°K. the density of thallium is 11.18 g./cc. From Equation (19) and the value of δ the reduced volume is calculated to be 0.78. Therefore $\eta^* = 5.00$. From Equation (17) the viscosity at this temperature is calculated to be 2.36 centipoise. This value is to be compared with the experimental value 2.11 centipoise; the error amounts to about 12%. Although this error is greater than the estimated experimental error of 0.05 to 0.1 centipoise, it is seen that the correlation predicts a value which is quite acceptable for engineering purposes.

The viscosity of thallium has been estimated from the correlation over the experimental temperature range, and the predicted curve is compared with the experimental points in Figure 7. It is seen that although the predicted values are consistently on the order of 10% high, the predicted temperature dependence is quite satisfactory. These results are encouraging because most of the remaining metals whose viscosities have not been measured melt at temperatures well above 1,000° or 1,500°K. At these

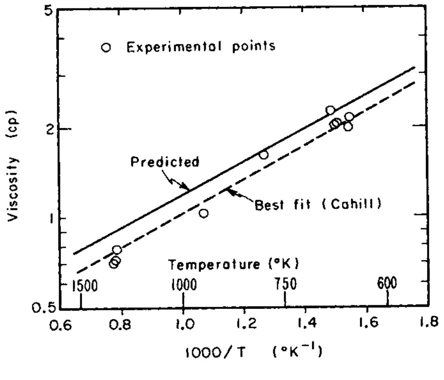

Fig. 7. Predicted viscosity of liquid thallium compared with the experimental data of Cahill and Grosse (*24*).

high temperatures experimental difficulties become considerable, and experimental errors of 10% and much more are to be expected. Thus it is felt that as experimental conditions become more severe, the predictive ability of this correlation becomes increasingly valuable.

DISCUSSION

It has been seen that some rather crude assumptions about the behavior of the microscopic liquid structure under nonuniform conditions have made it possible to develop the formal and rigorous, though incomplete, theory of liquids into a corresponding states correlation of viscosity for engineering use. An important consequence of this development is that it provides a set of consistent effective atomic interation energy parameters for a large number of metals. The success of the two parameter potential function, approximate though it may be, in correlating the viscosities indicates that corresponding states correlations of other properties of the metals might be established on the basis of these parameters. For instance, the parameters determined here have been used successfully by the author to correlate the configurational heat capacities of several liquid metals and to explain the behavior of monatomic liquid heat capacity with temperature (*25*).

Furthermore, since the analysis has led to an expression for liquid viscosity which takes into account the fundamental features of liquid structure and molecular behavior which appear to be responsible for the macroscopic phenomena, it is suggested that similar consideration of the effect of a flow field on the structure of a liquid mixture could lead to an appropriately modified form of Equation (15) for the viscosity of a mixture. Such an equation might indicate a method of correlating the viscosities of molten alloys with the use of the pure substance energy parameters determined here.

In any case it is felt that the general theory based on the pair distribution function provides the most meaningful approach to the problem of liquid properties now available and that consideration of its implications leads to a better understanding of the nature of irreversible processes in liquids.

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NOTATION

A_i = constant coefficient in function $F_i(r)$
C_i = constant coefficient in function $G(T^*)$
D = self-diffusion coefficient, sq. cm./sec.
$F(T^*, V^*)$ = universal function
$F_i(r)$ = coefficients in perturbation series
$f(\xi)$ = velocity distribution function
$G(T^*)$ = universal function
$g(\mathbf{r})$ = pair distribution function
$g^o(r)$ = equilibrium pair distribution function, radial distribution function
$g'(\mathbf{r})$ = deviation of pair distribution function from equilibrium
I = unit tensor
k = Boltzmann constant
M = molecular weight
m = mass of atom
N_o = Avagadro's number
n = particle number density
P = pressure tensor
p = thermodynamic pressure
R = gas constant
$\mathbf{r}$ = radial position vector
r = radial component of position vector
s = perturbation parameter
T = temperature, °K.
$\mathbf{u}$ = macroscopic velocity vector
u_δ = average macroscopic speed at $r = \delta$
u^* = average thermal speed of atoms
V^* = reduced volume
x = reduced radial distance

Greek Letters

δ = distance parameter characteristic of pair potential energy function and radial distribution factor
ϵ = energy parameter characteristic of pair potential energy function
η = viscosity, centipoise
η^* = reduced viscosity
θ, ϕ = angles representing $\mathbf{r}$ in spherical coordinates
κ = subscript indicating kinetic term
$\tilde{\xi}$ = microscopic velocity vector
τ = stress tensor
$\phi(r)$ = pair potential energy function

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